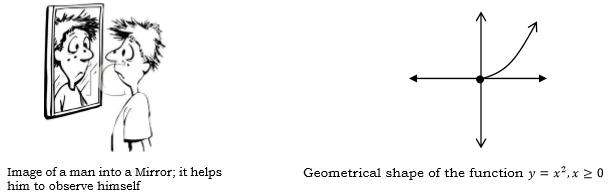
**Functions and their graphs**

**Introduction:** A function is an activity (work) but the graph is its reflection. A function is, so to say, completely observed only through its graph as we see that a man’s image is clearly reflected by a mirror. In mathematics the graph of a function is the geometrical representation (visual form) of its equation. In physics the same thing is called the wave which as for the musician is the representation of a sound that a sound source makes.



**Function:** If a variable *y* depends on a variable *x* in such a way that each value of *x* determines exactly one value of *y*, then *y* is called a function of *x* and it is denoted by the following symbol,



where *x* is independent variable and *y* is dependent variable. The inverse of this function is denoted by .

**Example:**;;; etc.

Alternatively, let  and  be two non empty sets. A mapping is called function if each element in  is assigned to unique element in .

**Types of functions:** There are many types of functions. These have been discussed as:

**Single valued function:** A function is called a single valued function if there exist only one value of *y* for each value of *x*.

**Example:**;;; etc.

**Many valued function:** A function is called a many valued function or multiple valued function if there exist more than one value of *y* for each value of *x*.

**Example:**;; etc.

**Algebraic function:** A function  which consists of a finite number of terms involving powers and roots of *x* is defined as an algebraic function.

**Example:**is an algebraic function.

**Polynomial Function:** A polynomial is an expression containing multiple terms with the operations of addition, subtraction, multiplication and degree of the each term is non-negative. A Function that consist with polynomial is called polynomial function.

**Example:** The function  is a polynomial function of order *n* whereand.

**Note:**

1. When then  is called polynomial function of order n.
2. When  then  is called monic polynomial function of order n.
3. When  then  is called polynomial function of order zero means constant polynomial function.
4. When  then  is called polynomial function of order one (1) means linear polynomial function.
5. When  then  is called polynomial function of order two means polynomial function of degree 2 or Quadratic polynomial function. The graph of a quadratic polynomial is a parabola.
6. When  then  is called polynomial function of order three means polynomial function of degree 3 or Cubic polynomial function.
7. When  then  is called polynomial function of order four means polynomial function of degree 4 or by-quadratic polynomial function.
8. When  then this types of polynomial is called zero polynomial with explicitly undefined degree. The graph of a zero polynomial is the x-axis.

Polynomials can be classified by the number of terms with nonzero coefficients, so that a one-term polynomial is called a [monomial](https://en.wikipedia.org/wiki/Monomial), a two-term polynomial is called a [binomial](https://en.wikipedia.org/wiki/Binomial_(polynomial)), and a three-term polynomial is called a *trinomial*. The term "quadrinomial" is occasionally used for a four-term polynomial. A polynomial in one variable is called a *[univariate](https://en.wikipedia.org/wiki/Univariate" \o "Univariate) polynomial*, a polynomial in more than one variable is called a multivariate polynomial. A polynomial with two variables is called a bivariate polynomial.

**Linear polynomial function:** A polynomial function in which degree/ order of the leading term is exactly one is called linear polynomial function.

**Example:**is a linear polynomial function with single variable *x*.

**Quadratic polynomial function: Case 01:** A polynomial function of the form  is called quadratic polynomial function which represents a parabola. When the value of “a “is positive then the parabola is concave up/open upward and otherwise concave down/open downward. The vertex of the parabola is . In another way we get value of the ordinate of vertex by putting the value of abscissa  in the equation .

**Case 02:** A polynomial function of the form  is called quadratic polynomial function that represents geometrically a parabola. When the value of “a “is positive then the parabola is open right parabola and otherwise it is open left parabola. The vertex of the parabola is  . In another way we get value of the abscissa of vertex by putting the value of ordinate  in the equation .

**Rational function:** A function of the form where  and both are the function of *x* and also  is called a rational function.

**Example:** The function is a rational function in single variable *x*.

**Transcendental function:** Functions that can’t be expressed as algebraic functions are called transcendental functions. These functions are of the following types:

1. **Exponential function:** A function of the form  is called an exponential function with base *b*.

**Examples:**, ,  , etc.

1. **Logarithmic function:** A function of the form  is called a logarithmic function with base *b*.

**Examples:**, , etc.

1. **Trigonometric function:** Functions of the types  are called trigonometric functions.
2. **Inverse trigonometric functions:** Functions of the types ,are called inverse trigonometric functions.

**Hyperbolic function:**

**Explicit function:** When a relation of two variables *x* and *y*is expressed as where*y* can be expressed directly in terms of *x*, then *y* is called an explicit function of *x*.

**Example:** is an explicit function of *x*.

**Implicit function:** When a relation of two variables *x* and *y* is expressed as, where*x*andy cannot be expressed directly in terms of the other, then either variable is called an implicit function of the other.

**Example:** is an implicit function.

**Even function:** A function is called an even function if it satisfies the condition

.

**Example:** are even functions.

**Odd function:** A function is called an odd function if it satisfies the condition

.

**Example:** are odd functions.

**Periodic function:** A function is called a periodic function of period ***T*** if it satisfies the condition .

**Example:** (1). are periodic function of period 2π.

(2). are periodic function of period π.

**Absolute value function:** A function is called an absolute value function.

**Example:** is an absolute value function.

**Bounded function:** A function defined on an interval , is called a bounded function if there exists a number *M* such that .

**Or,** A function is called a bounded function if its range is a bounded set.

**Example:**  is a bounded function.

**Increasing function:** A function defined on an interval where , is called an increasing function over the interval if .

**Example:** is an increasing function.

**Decreasing function:** A function defined on an interval where , is called a decreasing function over the interval if .

**Example:** is a decreasing function.

One-one function:

Onto function:

Constant function:

**Compositions of functions:**

**Inverse function:**

**Domain:** The set of all values of*x* for which the function is defined, is called domain of the function. Simply domain is the set of all allowable *x*-values.

Mathematically, .

**Range:** The set of all values of *y* corresponding to the *x* values for which the function is defined, is called range of the function. Simply range is the set of all possible y-values.

Mathematically, .

**Interval:** If the set of all real numbers lie between two real numbers *a* and *b*, where then the set of all real numbers is called an interval.

Intervals are four kinds:

1. The set is called a closed interval, denoted by .
2. The set is called an open interval, denoted by .
3. The set is called a left half open interval, denoted by .
4. The set is called a right half open interval, denoted by .

**Problem 01:** Find the domain and range of the function.

**Solution:** Given function is,



Here, *y* gives real values for all real values of *x*.

So, the domain of the given function is,



**Again,** we have,







Here, *x* gives real values for all real values of *y*.

So, the range of the given function is,

(Ans)

**H.W:**

Find the domain and range of the following functions

1.Ans:  and 

2. Ans:  and 

3. Ans:  and 

**Problem 02:** Find the domain and range of the function.

**Solution:** Given function is,



Here, *y* gives real values for all real values of *x*.

So, the domain of the given function is,



**Again,** we have





In the above equation the values of *x* will be real if and only if its *Discriminant.*

 ; []











Therefore the range of the given function is,

(Ans)

**Alternative way, For range** we have















Here, x is defined if





Therefore the range of the given function is,

(Ans)

**H.W:**

Find the domain and range of the following quadratic functions

1. Ans:  and 

2. Ans:  and 

4. Ans:  and 

5.Ans:  and 

6. Ans:  and 

7. Ans:  and 

**Problem 03:** Find the domain and range of the function.

**Solution:** Given function is,



Here, y is undefined if





So, y gives real values for all real values of x except.

Therefore, the domain of the given function is

.

Again we have,











Here, x is undefined if





So, *x* gives real values for all real values of y except.

Therefore, the range of the given function is

 (Ans)

**Problem 04:** Find the domain and range of the function.

**Solution:** Given function is,



Here, y is undefined if





So, y gives real values for all real values of x except.

Therefore, the domain of the given function is

.

Again we have,









Here, *x* is defined for all real values of *y* except 

Therefore, the range of the given function is

 (Ans)

**H.W:**

Find the domain and range of the following quadratic functions

1. Ans:  and 

2. Ans:  and 

3.Ans:  and 

4.Ans:  and 

5. Ans:

**Problem 05:** Find the domain and range of the function.

**Solution:** Given function is,



Here, y gives real values iff







Therefore, the domain of the given function is

.

Again,



The values of *y* in (1) are positive or zero, *i.e,*.

Now;. [Squaring both sides]

 ;.

 ;.

 ;.

Here, *x* is defined for .

Therefore, the range of the given function is



***(Ans).***

**Problem 06:** Find the domain and range of the function.

Solution: Given function is,



Here, y gives real values iff









Therefore, the domain of the given function is

.

**Again,** we have,



The values of y in (1) are negative or zero, *i.e*,.

Now [Squaring both sides]







Here*, x* is defined for .

Therefore, the range of the given function is



**(Ans).**

**H.W:**

Find the domain and range of the following functions

1. Ans:  and 
2. Ans:  and 
3. Ans:  and 
4. Ans:  and 
5. Ans:  and 
6. Ans:  and 
7. Ans:  and 

**Problem 07:** Find the domain and range of the function.

**Solution:** Given function is,



Here, *y* gives real values iff,









This inequality is satisfied if



Therefore, the domain of the given function is,







**Again,** we have,



The values of *y* in (1) are positive or zero *i.e, ,*

Now,  [Squaring both sides]





In the above equation the values of *x* will be real if and only if it’s *Discriminant.*

*i.e,*[]









Here, *x* is defined for .

So the range of the given function is



**(Ans).**

**Problem 08:** Find the domain and range of the function.

**Solution:** Given function is,



Here, y gives real values iff,



This inequality is satisfied for all real values of *x*.

Therefore the domain of the given function is,

.

**Again,** we have,

… … (1)

The values of *y* in (1) are positive and lowest value is 1,*i.e,*

Now  ; [Squaring both sides]

 ;

 ;

In the above equation the values of *x* will be real if and only if its *Discriminant.*

i.e,;[]

;

;

;

Here, *x* is defined for all 



**(Ans).**

**Problem 09:** Find the domain and range of the function.

**Solution:** Given function is,



Here, y gives real values iff,





This inequality is satisfied if,



Therefore, the domain of the given function is,





**Again,** we have,

… … (1)

The values of *y* in (1) are positive and lowest value is zero, *i.e,*.

Now  ; [Squaring both sides]

 ;

 ;

In the above equation the values of *x* will be real if and only if it’s *Discriminant.*

i.e,  ;[]

 ;

 ; [Dividing by -4]

Here, *x* is defined for all 

Therefore the range of the given function is,



 (Ans.)

**H.W:**

Find the domain and range of the following functions

1. Ans:  and 
2. Ans:  and 
3. Ans:  and 
4. Ans:  and 
5. Ans:  and 
6. Ans:  and 
7. Ans:  and 
8. Ans:  and 

**Problem 10:** Find the domain and range of the function.

**Solution:** Given function is,



Here, *y* gives real values iff,







Therefore the domain of the given function is.



**Again**, we have,



The values of *y* in (1) are positive and lowest value is near to 0,*i.e,*.

Now,  ;

 ;

 ;

 ;

Here, *x* is defined for all .

Therefore the range of the given function is



**(Ans)**

**Problem 11:** Find the domain and range of the function.

**Solution:** Given function is,



Here, *y* gives real values iff,



This inequality is satisfied if or.

Therefore the domain of the given function is,

.

.

**Again,** we have,



The values of *y* in (1) are positive or zero,*i.e,*.

Now, ; [Squaring both-sides]

 ;

 ;

 ;

 ;

 ;

 ;

Here, *x*is defined for allexcept.

So, the range of the given function is



**(Ans.)**

**Problem 12:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values for all real values of *x*.

So, the domain of the given function is,



**Again,** we have,







Here, *x* gives real values iff.

So, the range of the given function is,



**(Ans).**

**Problem 13:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values iff



This inequality is satisfied if.

So, the domain of the given function is,



**Again,** we have,













Here, *x* gives real values for all real values of

So, the range of the given function is,





**(Ans).**

**Problem 14:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values for all real values of *x*.

So, the domain of the given function is,



**Again,** we have,





Here, *x* gives real values for

So, the range of the given function is,



**(Ans).**

**Problem 15:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values for all real values of *x* except 

So, the domain of the given function is,



**Again,** we have,





Here, *x* gives real values for all real values of y.

So, the range of the given function is,





**(Ans).**

**Problem 16:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values for all real values of x except 

So, the domain of the given function is,



**Again,** we have,





Here, *x* gives real values for all real values of y.

So, the range of the given function is,





**(Ans).**

**Problem 17:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values for all real values of x except 

So, the domain of the given function is,



**Again,** we have,





Here, *x* gives real values for all real values of y except .

So, the range of the given function is,



**(Ans).**

**Problem 18:** Find the domain and range of .

**Solution:** Given function is,



Here, *y* gives real values for all real values of x except 

So, the domain of the given function is,



**Again,** we have,





Here, *x* gives real values for all real values of y except .

So, the range of the given function is,



**(Ans).**

**H.W:**

Find the domain and range of the following functions:

1. 
2. 
3. 

**Graph of Functions**

If denotes a function, then the graph of the function is the set of all ordered pairs for all values of *x* in the domain A.

Graph of

Therefore, Graph is the geometrical/Pictorial representation of a function or visualization of a function.

**Graph of some elementary functions:**

* ***Graph of +c***

Y

X

**m< 0**

O

X

Y

**m> 0**

O

* ***Graph of***

Y

X

O

* ***Graph of*** *⯂****Graph of***

Y

X

O

Y

X

O

* ***Graph of*** *⯂****Graph of***

Y

X

O

Y

X

O

* ***Graph of*** *⯂****Graph of***

Y

X

O

Y

O

X

***Note:*** *when power of the variable increases then graph will be wider.*

* ***Graph of*** *⯂****Graph of***

O

X

O

X

Y

Y

* ***Graph of*** *⯂****Graph of***

O

X

Y

O

X

Y

* ***Graph of*** *⯂****Graph of***

O

X

Y

O

X

Y

* ***Graph of*** *⯂****Graph of***

O

X

Y

O

X

Y

O

X

Y

(0,1)

* ***Graph of*** *⯂****Graph of***

O

X

Y

O

X

Y

* ***Graph of*** ⯂***Graph of***

O

X

Y

(1,0)

(0,1)

* ***Graph of*** *⯂* ***Graph of***

O

X

Y

(0,1)

O

X

Y

(0,1)

* ***Graph of*** *⯂****Graph of***

O

X

Y

-a a

O

X

Y

(1,0)

* ***Graph of*** *⯂****Graph of***

O

X

Y

-a a

O

X

Y

- aa

* ***Graph of*** *⯂****Graph of***

O

X

Y

-a a

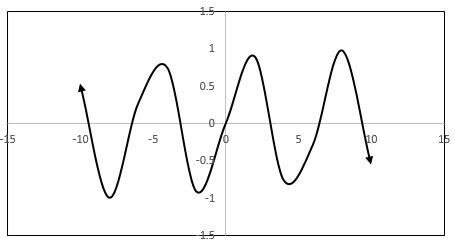
-bb

O

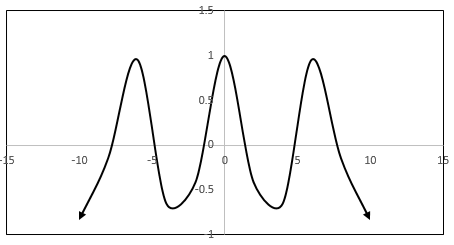
X

Y

* ***Graph of***



* ***Graph of***



**Transformation of Function**

Transformation of a function is any kind of change in the function such as move or resize the graphs of functions. There are two types of transformation of thefunctions such as,

1. ***Translation/Shifting:***Any kind of shifting of the graph of a function is called

translationof the function that means changing the location of the graph without changing its size and shape is called translation.

1. ***Scaling:***Scaling of a graph of a function is a transformation in which the size and shape of the graph is changed.

* ***Translation***

***Horizontal translation:***

*Function:*

*Forthe graph is translated c units to the left.*

*Forthe graph is translated c units to the right.*

***Vertical Translation:***

*Function:*

*For the graph is translated c units upward.*

*For the graph is translated c units downward.*

* ***Scaling***

*Function:*

*For the graph is compressed.*

*Forthe graph is stretched.*

**Problem- 01:** Sketch the graph of the function.

**Solution:** The equation of the given function is,



Completing the given equation in a square form it becomes as







**X**

**Y**

**O**

The graph of the standard function is as follows

**-3**

**X**

**Y**

**O**

Translating or shifting the above graph 3 units to the left, we get the graph of the function.



**-3**

**X**

**Y**

**O**

Translating or shifting the above graph 1 units upward, we get the graph of the function.



**(Desired Graph)**

**H.W:**

Sketch the graph of the following functions

1. **4.**
2. **5.** 
3. **6.**

**Problem -02:** Sketch the graph of the function.

**Solution:** The equation of the given function is,



The graph of the standard positive square root function  is as follows

**X**

**Y**

**O**

Translating or shifting the above graph 2 units to the right, we get the graph of the function.

**X**

**Y**

**O**

**2**

Translating or shifting the above graph 5 units upward, we get the graph of the function.

**X**

**Y**

**O**

**2**

**5**

**(Desired Graph)**

**H.W:**

Sketch the graph of the following functions

1. **5.**
2. **6.** 
3. **7.** 
4. **8.**

**Problem -03:** Sketch the graph of the function.

**Solution:** The equation of the given function is,



The graph of the standard absolute value function  is as follows

**X**

**Y**

**O**

Therefore the graph of the standard absolute value function  is as follows

**X**

**Y**

**O**

Translating or shifting the above graph 2 units to the left, we get the graph of the function.

**X**

**Y**

**O**

**-2**

Translating or shifting the above graph 2 units upward, we get the graph of the function or.

**X**

**Y**

**O**

**-2**

**2**

**(Desired Graph)**

**H.W:**Sketch the graph of the following functions:

1. 
2. 
3. 

**Piecewise function:**A **piecewise-defined function** (also called a **piecewise function** or a **hybrid function**) is a [function](https://en.wikipedia.org/wiki/Function_(mathematics)) which is defined by multiple sub-functions, each sub-function applying to a certain interval of the main function's domain (a sub-domain).

**For example:** The following function is the piecewise function



**Note that:**

1. The function  is defined on the interval.
2. The function  is defined on the interval.
3. The function  is defined on the interval.

**Mathematical Problem**

**Problem -01:**Sketch the graph of the function . Also find domain and range of the function.

**Solution:**Given function is



In the interval  or, the graph of the functionis,

**X**

**O**

**Y**

**-1**

In the interval or, the graph of the function  which is an upper semi-circleof radius 1 units and Centre at origin is,

**X**

**Y**

**O**

**1**

**-1**

In the interval  or, the graph of the function is,

**X**

**Y**

**O**

**1**

Therefore, the graph of the given function is as follows:

**X**

**Y**

**O**

**1**

**-1**

**(Desired Graph)**

**Again,**the domain is,







And the range is,



**(Ans.)**

**Problem -02:**Sketch the graph of the function.Also find domain and range of the function.

**Solution:**Given function is,



In the interval  or, the graph of the function is,

**X**

**Y**

**O**

In the interval  or, the graph of the function is,

**X**

**Y**

**O**

**1**

In the interval  or, the graph of the function is,

**X**

**Y**

**O**

**1**

Finally, the graph of the given function is as follows,

**X**

**Y**

**O**

**1**



**(Desired Graph)**

**Again,** the domain is, 





And the range is, 

**(Ans.)**

**Problem -03:**Sketch the graph of the function . Also find domain and range of the function.

**Solution:**Given function is,



In the interval  or, the graph of the functionis,

**X**

**Y**

**O**

**1**

In the interval  or, the graph of the functionis,

**X**

**Y**

**O**

**1**

**1**

In the interval or, the graph of the functionis,

**X**

**Y**

**O**

**1**

Finally, the graph of the given function is as follows:

**X**

**Y**

**O**

**1**

**1**

**(Desired Graph)**

**Again,** the domain is,







And the range is,



**(Ans.)**

**H.W:** Sketch the graph of the following piecewise functions:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 

**Modulus/absolute function:**The modulus or absolute value of *x* is denoted by the symbol and is defined as follows,



Geometrically the modulus or absolute value of a number represents the distance of that number from the origin. The absolute value of *x* is always positive or zero.

A function together with modulus or absolute value sign is called modulus function.

**For example:** The function is an absolute value function or Modulus function.

**Breaking point of a function:**Breaking point of a function is a point at which the function changes.

**For example:** The function  has two breaking points are.

**Procedure of Graphing Absolute value function:**

1. At first convert the modulus function into piecewise function according to its number of breaking points.

2. After that sketch the graph as piecewise function.

**Mathematical Problem**

**Problem -01:**Sketch the graph of the function.

**Solution:**Given absolute value function is,



For breaking points  and.

**1**

**0**

There are two breaking points in this mathematical problem such as &and these points divide real number line into three intervals.Therefore, we define this absolute value function section-ally by three parts.

Now, 





**Graph:** In the interval or, the graph of the functionis,

**X**

**Y**

**O**

**1**

In the interval  or, the graph of the functionis,

**X**

**Y**

**O**

**1**

**1**

In the interval or, the graph of the functionis,

**X**

**Y**

**O**

**1**

Finally, the graph of the given function is as follows:

**X**

**Y**

**O**

**1**

**1**

**(Desired Graph)**

**Problem -02:**Sketch the graph of the function.

**Solution:**Given absolute value function is,



For breaking points  andan also 

**-1**

**1**

**0**

There are three breaking points in this mathematical problem such as &and those points divide real number line into four intervals.Therefore, we define this absolute value function section-ally by four parts.

Now,







**Graph:** In the interval  or, the graph of the functionis,

**X**

**Y**

**O**

**1**

**3**

In the interval or, the graph of the functionis,

**X**

**Y**

**O**

**1**

**3**

In the interval or, the graph of the functionis,

**X**

**Y**

**O**

**-1**

**3**

In the interval  or, the graph of the functionis,

**X**

**Y**

**O**

**-1**

**3**

Finally, the graph of the given function is as follows:

**X**

**Y**

**O**

**1**

**3**

**-1**

**(Desired Graph)**

**H.W:**

Sketch the graph of the following absolute value functions:

1. 
2. 
3. 
4. 
5. 
6. 